DOCUMENT RESUME

ED 242 535 SE 044 295

AUTHOR Quintero, Ana Helvia
TITLE Children's Difficulties with Two-Step Word Problems.
PUB DATE Apr 84
NOTE 23p.; Paper presented at the Annual Meeting of the American Educational Research Association (68th, New Orleans, LA, April 23-27, 1984).
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Educational Research; *Elementary School Mathematics; *Error Patterns; Grade 5; Intermediate Grades; *Mathematics Instruction; *Problem Solving
IDENTIFIERS *Mathematics Education Research; Puerto Rico; *Word Problems (Mathematics)

ABSTRACT

This study focused on analyzing children's difficulties with two-step mathematical word problems. Seventy-one fifth-grade children in Puerto Rico were individually observed solving five problems. Two of these were two-step problems; the remaining three were one-step problems with the same mathematical structures as the components of the two-step problems. Performance was significantly poorer on the two-step problems than on the one-step problems. Forty-eight (66.6%) children had difficulties with the two-step problems. The difficulties stemmed from different sources: lack of understanding of the concepts and relationships involved in the problem; incorrect strategy for working any word problem; using a single operation to solve two-step problems; and difficulties with a mismatch between order of presentation and order of operation. The implications for education of these results are briefly discussed. (Author/MNS)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED242535

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

✓ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official NIE position or policy

Children's Difficulties with Two-Step Word Problems

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Ana Helvia Quintero

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

By

Ana Helvia Quintero
Associate Professor
Department of Mathematics
University of Puerto Rico
Río Piedras, Puerto Rico

Paper accepted for presentation at the American Educational Research Association Annual Meeting, April 1984, New Orleans, Louisiana.

SE044295

Abstract

This study analyzes children's difficulties with two-step mathematical word problems. Seventy-one fifth grade children were individually observed solving two-step problems. The children also solved one-step problems with the same mathematical structures as the components of the two-step problems.

Forty-eight (66.6%) children had difficulties with the two-step problems. The difficulties, however, stem from different sources: lack of understanding of the concepts and relationships involved in the problem; incorrect strategy for working any word problem; using a single operation to solve two-step problems; difficulties with a mismatch between order of presentation and order of operation.

The implications for education of these results are also discussed.

Children's Difficulties with Two-Step Word Problems

In the elementary grades mathematical word problems that can't be solved by a routine application of a single arithmetic operation cause a great deal of difficulty (see for example National Assessment of Educational Progress, 1979; Carpenter, Corbitt, Kepner, Lindquist and Reys, 1980; Zweng, 1979). It is important to identify why are these non-routine problems so difficult.

Goldin and Mc Clintock (1979) have identified a set of task variables that affect mathematical problem solving: syntax variables, content and context variables; heuristic behavior variables; structure variables. It is important to study the role of each of these variables in children's behavior when solving non-routine problems. This work will study the role of these variables when children solve two-step problems of a given type.

In a previous study Quintero (1983) found that a source of difficulty in solving two-step problems with the structure "a x's for b y's and c y's" (for example: "6 lollipops for every 2 chocolate bars and bought 8 chocolate bars") was a lack of understanding of the concepts and relationships involved in the problems. She also found that some children understood the concepts and relationships in the problem and still worked the problems incorrectly. This group of children could be having difficulties with the strategy of solving two-step problems. Yet, it was not clear from the study the specific nature of these difficulties. This study will then emphasize the nature of children's difficulties with the strategy of solving two-step problems. Seventy-one children were observed solving two-step problems. A set of one-step problems corresponding to the components of the two-step problems were introduced so as to probe children's

difficulties with the concepts and relationships involved in the problem. If a group of children could solve the one-step problems but not the two-step problem involving the same concepts then they probably are having difficulties with the strategy of solving the problem. The pattern of error of this group of children could shed light on children's incorrect strategies.

EXPERIMENT I

Subjects

The study was carried out with 71 children from the fifth grade of the University Elementary School of Puerto Rico. Their ages range between 9 and 11 years old (9 years N =11; 10 years N = 49; 11 years N = 11). There were 30 boys and 41 girls.

Procedures and materials

The expermienter worked with one child at a time. The session with each child lasted from 10 to 20 minutes. The child solved 5 word problems. Two were two-step problems; one of these was solved by multiplying and then adding; the other one was solved by multiplying and then subtracting. In the former the order of presentation coincides with the order of working the operation; in the latter the order of presentation was different to the order of working the operations. The three quantities used to solve the two-step could be expressed in numeral (e. g. 7 years) or in words (three times as old). In one problem the three quantities were expressed in numerals while in the other only two quantities were expressed in numerals.

The rest of the problems could be solved in one-step. They had the same mathematical structure as the components of the two-step problems.

For example, one of the two-step problems was: [1]

"Ricardo is 7 years old.

Pedro is 5 times as old as Ricardo.

Juan is 4 years older than Pedro.

How old is Juan?"

and the one-step problems corresponding to its component were:

"Ana has 9 lollipops.

Carmen has 5 times as many.

How many lollipops does Carmen have?"

which has the same structure as the first part of the former problem; and

"María is 14 years old.

José is 5 years older than María.

How old is José?

which is equivalent to the second part of the two-step problem.

Results Experiment I

Performance

The performance of the children on the two-step problems was significantly poorer than their performance on the one-step problems (Chi Square Test $p \leq .001$). Among the two-step problems, the one where the order of presentation coincided with the order in which the operation was performed was easier than the other two-step problem (Chi Square Test $p \leq .10$). The difference on performance among the different types of one-step problems was minimal (see table 1).

Insert Table 1 around here

Thirty-three out of forty eight times (46.47% of the total, 68.75% of those making mistakes) that the two-step problems were worked

incorrectly; the one-step problems corresponding to their components were worked correctly (see table 2).

Insert table 2 around here

Type of errors

Children made different types of errors when solving the two-step problems:

(1) Working the problem as a one-step problem. For example, in the above two-step problem multiplying $\begin{array}{r} 7 \\ \times 5 \end{array}$ or adding $\begin{array}{r} 5 \\ +4 \end{array}$ but not both.

(2) Adding or multiplying all the numbers that appear in the problem. In the above example:

$$\begin{array}{r} 7 \\ 5 \\ +4 \\ \hline 16 \end{array} \qquad \text{or} \qquad \begin{array}{r} 7 \\ \times 5 \\ \hline 35 \\ \times 4 \\ \hline 140 \end{array}$$

(3) Working one of the steps incorrectly when solving the problem in two steps. In the above example:

$$\begin{array}{r} 7 \\ \times 5 \\ \hline 35 \\ -4 \\ \hline 31 \end{array}$$

Discussion Experiment I

A considerable number of children (48 out of 71, 66.6%) had difficulty with at least one of the two-step problems. Children's pattern of errors suggest that their difficulties with two-step problems stem from different sources.

Some children (N = 19; 26.7% of the total; 39.5% of the ones making mistakes) worked incorrectly at least one of the one-step problems

corresponding to a component of the two-step problem worked incorrectly (this group of children will be denoted as "children working incorrectly the components"). Most of the time (12 out of 19, 63%) the type of error in the one-step problem corresponded to the type of error in the two-step problem. For example, a boy (10 years old) in the one-step multiplication problem added "9 + 5" instead of multiplying them. In the two-step problems he also added "7 + 5" when he should have multiplied them.

Some of these children could be having difficulties in understanding the concepts involved in the two-step problems or in associating the correct mathematical operation with the concept at hand. For example, it was observed that seven children added whenever the word "times" ("veces" in Spanish) appear.

Another observation that points to possible conceptual difficulties is that two other children when explaining their method of solving a problem (worked incorrectly), changed what the problem said. In both cases the new version of the problem corresponded to the operation they used to solve the problem. It seems, then, that the way they restated the problem was the way they understood it.

Children working incorrectly the components may also have other types of difficulties. One of them could be using an incorrect strategy for solving word problems. For example, one girl (10 years old) consistently added the numerals that appeared in all the problems. In the two-step problem where two of the quantities were expressed in numerals she added these two quantities; when all the quantities were expressed in numerals she added three quantities. She added, even, in the one-step subtraction problem. This girl seems to have the following strategy for solving any problem: add all the numerals that appear in the problem statement.

As with this girl, other children working the components incorrectly are probably using a common incorrect strategy when solving one-step and two-step problems.

A group of children (N = 29, 40,8% of the total; 60.4% of the ones making mistakes in the two-step problems) seem to be having difficulties in the method used to solve two-step problems. These children worked correctly each of the one-step problems corresponding to the components of the two-step problem worked incorrectly so they understand the concepts involved in the two-step problem and use a correct strategy for solving one-step problems. Their difficulty must lie in the strategy used for solving two-step problems.

Analyzing the patterns of error of this group of children we identify a variety of incorrect strategies. One of them can be inferred from the differences in performance between the two two-step problems. The performance of this group of children on the two-step problem whose order of presentation coincides with the order of working the operation was better than their performance on the two-step problem where there was a mismatch between order of presentation and order of operation. Among the children who have worked incorrectly the components the difference on performance between the two two-step problems was minimal.

Insert table 3 around here

These facts suggest that a group of children have difficulties dealing with a mismatch between the order that the information is presented and the order in which it is used. A possible strategy used by this group of children is the following.

Insert Diagram 1 around here

The strategy described in Diagram I leads to solve correctly the problem when the order of presentation and operation coincide, but when there is a mismatch it leads to error.

Another incorrect strategy used to solve the two-step problems was using a single operation to solve the problem. This was the most common mistake when solving the two-step problems. Some children applied the single operation to two numerals (for example $\begin{array}{r} 7) \\ \times\ 5 \\ \hline 35 \end{array}$); others apply the single operation to all the numerals (e. g. $\begin{array}{r} 7) \\ 5 \\ +\ 4 \\ \hline 16 \end{array}$).

These two types of mistake could be related to the experience of solving word problems in school. Usually elementary school problems are one-step problems. Children could then interpret problem-solving as simply choosing the appropiate operation.

Several observations made during this study support the above statement. For example, a child who had worked a two-step problem as a one-step problem was asked to explain her answer. As she was reading the problem again she said:

S: "Wait a moment, I worked this problem incorrectly"
(She works the problem again this time correctly)

E: "Why did you make that mistake?

S: "I paid too much attention to the question and to the numbers and not to what the problem says"

The comments of this girl suggest a possible method used by the children to solve the word problems. They first look at the question. Then following the same pattern of solving word problems that they used

for solving one-step problems they search for information in the problem statement to find which operation should be used to answer the question. As soon as they find some relevant information they do the corresponding operation. For example if they first find "he is 4 years older" they perform addition, on the other hand if they find "he is 5 times as old" they multiply. In fact, all the children who solved correctly the components and work the two-step problem using a single operation, used one of the arithmetic operation that is required to solve the two-step problem.

Another possible strategy associated with school experience is similar to the one described above. The student search the problem to find what operation must be done. Once they have identified the operation instead of applying it to the numerals associated to the operation, they applied it to all the numerals in the problem.

Zweng (1979) results give support to the possibility that a group of children is using the strategies described in the previous paragraph. She found that performance on one-step problems improved with school grade. Yet, on multistep problems this was not the case. As the following table shows the performance of students tends to get worst as the student progress on school grade. One can infer from these

Insert table 4 around here

results that as students get more practice in word problems in school (problems which are generally one-step) strategies related to one-step problems become more readily available. They tend then to use these strategies in any new situation in which they are asked to solve problems.

If this is the case then if children are told beforehand that the problems given to them are solved using two operations their performance should improve. Experiment II will probe this hypothesis.

Experimen. II

Subjects

The children in this study had already participated in Experiment I. They were the last 22 children who worked incorrectly at least one two-step problem while working correctly its componet. Their ages range between 9 and 11 years old (9 years N = 2; 10 years N = 15; 11 years N = 5). They were 14 girls and 8 boys.

Procedures and materials

Once the children had solved the problems of Experiment I they were asked to solve two additional two-step problems with the same mathematical structure as the two-step problems that they had just solved. For example in Experiment I they solved:

"Ricardo is 7 years old.

Pedro is 5 times as old as Ricardo.

Juan is 4 years older than Pedro.

How old is Juan?"

The extra problem corresponding the above was:

María has 7 pieces of candy.

Carmen has 5 times as many as María

Laura has 4 more pieces than Carmen.

How many pieces of candy does Laura have?

Half of the children were told that the extra problems were solved using two operation (this group of children will be called Group 1). The rest of the children solved the extra problems as part of the original task (Group 2).

Results

A total of 9 children (40.9%) improved their performance on the two extra problems. Six of these were in Group 1 and 3 were in Group 2. None of the children who worked correctly a two-step problem in the original task worked incorrectly the correspondent extra problem.

Discussion on Experiment II

Six out of eleven (54.5%) children improved their performance in the two-step problems after been told that the problems were solved using two operations. Three out of eleven (27%) children improved their performance by just working the problems again.

These results show that a group of children have the competence to work correctly the type of two-step problems used in this study but on some occasions they worked them incorrectly. It is our hypothesis that their performance is affected by their previous school experience. In the school curriculum word problems usually follow instruction about an arithmetic operation. Word problems are viewed as an area of application of children's knowledge about that operation. Students can then fix as strategy for working word problems looking for the appropiate operation. The results of Experiment 2 confirm this hypothesis. A group of children improved their performance while working two-step problems for a second time without any training. The fact that the improvement was greater when they were told that the problem was solved using two operations gives more weight to our hypothesis.

General discussion and conclusions

As I observed children solving problems in this study I was impressed by the great variety of ways that children worked, both correctly and incorrectly, the problems. This study's main interest was in identifying

what lead children to solve incorrectly two-step problems. Several types of difficulties were identified.

First there was a group of children who misunderstood the concepts or the way that you associate these concepts with mathematical operations. For example a group of seven children added whenever they had to multiply both in one-step and two-step problems.

Other errors were caused by using an incorrect strategy to solve word problems. Some of these strategies lead to error in both one-step and two-step problems. For example a girl solved every problem by adding all the numerals that appear in it. Other strategies lead to error only on two-step (or multi-step) problems. In this study we identified two such strategies.

One of these strategies permits the student to work two-step problems correctly if the order of presentation of the information coincides with the order in which you work the operation. If there was a mismatch among these two factors the problems were worked incorrectly.

A second type of incorrect strategy affects any two-step (or multi-step) problem. It consists of solving two-step problems using a single operation. It is our hypothesis that this strategy is rooted in children's experience solving word problems as an area of application of an operation just learned. They then see the task of solving problems as that of determining the operation that should be used. In solving word problems they search to find that operation and once they find some relevant information they perform a single arithmetic operation. If this hypothesis is true telling children that the problem at hand is solved using two operations should improve their performance. This was done in Experiment II and 54.5% of the children improved their performance.

If we consider the problem where the order of information coincides with the order of working the operation the percent of children that improved their performance goes up to 83.3%. In this problem the type of difficulty related to the mismatch between the order of presentation and the order of the operations is not present. So if the children do have an understanding of the concepts involved on the problem, the majority of their mistakes must be related to the strategy used of choosing a single operation.

On the other two-step problem, the one where there is a mismatch between order of presentation and order of operations, children's mistakes could be due to using either of the incorrect strategies related to two-step problems. In this problem children's performance shouldn't improve as much. This is what we found. Only 22.2% of the children improved their performance on this problem after being told that they needed two operations to solve the problem.

This work identified several factors that explain children's mistakes when solving two-step problems of a given type. Some of these factors affect performance on any word problem while others are specific to the problem being solved. The latter exemplifies the importance of having a clear description of the type of problem that is presented to the subject. This description must consider the different variables identified by Goldin and Mc Clintock (1979): syntax variables; content and context variables; structure variables; heuristic behavior variables. This study identified difficulties related to each of these variables; conceptual difficulties (content and context variables); strategies for solving any word problem, like add all numerals in the problem (heuristic behavior variable); working two-step problems using a single operation (structure and

heuristic behavior variables); difficulty with a mismatch between presentation and operation (syntax variables).

These results also show that the category used to characterize the problems given to the subjects shouldn't be too broad. If it is too broad it will probably include problems of various structures. Therefore, when looking for a general cause of difficulty we may overlook the ones which are related to some types of structures and not to others.

Educational Implications

To help children overcome their difficulties when solving word problems one has to identify the root of their mistakes.

This study has shown that children's errors solving two-step problems are due to different types of difficulties. Some of these difficulties are general to all word problems. Others are specific to the type of problem being solved. Given the great variety of word problems it is important to analize the problem children are given to solve in order to identify possible sources of mistakes. This research can help teachers in this task not only by pointing out possible areas of difficulties in a two-step problem of a given type, but also by suggesting a method that can be used by teachers in the classroom to identify children's difficulty with other types of word problems.

REFERENCES

Carpenter, T. P., Corbitt, M. K., Kepner, H. S. Jr., Montgomery Lindquist, M. and Reys, R. E., "Solving Verbal Problems: Results and Implication from National Assessment" Arithmetic Teacher, Vol. 28, No. 1, Sept. 1980: p. 8 - 12

Golding, G. A. and Mc Clintock, C. E. (ed), "Task Variables in Mathematics Problem Solving" The Georgia Center for the Study of Learning and Teaching Mathematics, Nov. 1979, ERIC ED, 178 - 366

National Assessment of Educational Progress, Changes in Mathematical Achievement, 1973 - 1978; Mathematical Applications, 1979

Quintero, A., "The Role of Conceptual Understanding in Solving Word Problems: Two-Step Problems"

Journal for Research in Mathematics Education, Volume 14, No. 2, March 1983, p. 102 - 112

Zweng, M., "Children's Strategies of Solving Verbal Problems" Final Report, National Institute of Education, 1979, ERIC ED, 178 - 359

Table 1

Performance on the word problems

	Correct	Incorrect
Two Step Problem (x, +) Same order of Operation	39	32
Two Step Problem (x, -) Different order of operation	29	42
One-Step (multiply)	58	13
One-Step (additon)	61	10
One-Step (subtract)	65	6

Table 2

Comparison of performance between two-step and one-step problems

Performance on the two-step problems	Performance on the one-step problems: All three one-step correctly	Two one-step problems correctly: +, x	Two one-step problems correctly: -, x	Two one-step problems correctly: +, -	One one-step problem correctly	None correctly
Both two-step problems incorrectly	8	2	5	8	2	1
One (x, +) two-step problem incorrectly	5	0	0	0	1	0
One (x, -) two-step problem incorrectly	12	2	1	1	0	0

Table 3

Comparison of performance between the two two-step problem

Problem	Group of children working the components (one-step problems) correctly		Group of children working the components (one-step prpblems) incorrectly	
	Correct	Incorrect	Correct	Incorrect
Order of presentation and performance coincide	14	14	2	18
Mismatch between presentation and performance	6	22	0	20

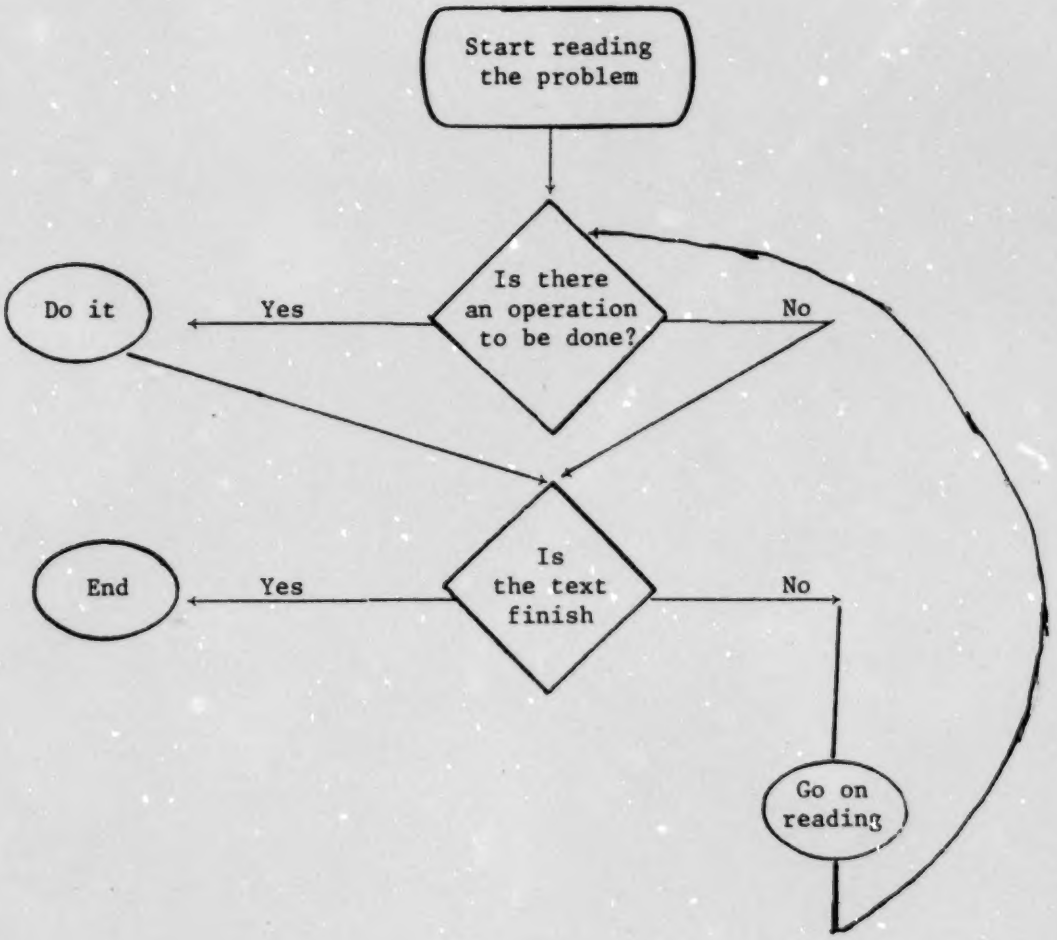
Table 4

Performance on Zweng's study on the multi-step problems (Percent correct)

Grade	Third	Fourth	Fifth	Sixth
Operations				
Multiplication	62	46	45	55
Division	57	58	40	37

Diagram 1

Strategy for solving two-step problems



Notes

(1) The original problems were in Spanish.